Przekształcenie Laplace'a jest to przekształcenie całkowe postaci

$$f \mapsto \mathcal{L}[f] = \overline{f},\ \text{gdzie } \mathcal{L}[f(t)](s) = \overline{f}(s) \stackrel{def}{=} \int_0^{+\infty} f(t)e^{-st}dt,\ s \in \mathbb{C}.$$

Własności przekształcenia Laplace'a

1. $\mathcal{L}[af_1 + bf_2] = a\mathcal{L}[f_1] + b\mathcal{L}[f_2],\ a, b \in \mathbb{R}$
2. $\mathcal{L}[f(t - t_0)] = e^{-t_0 s} \cdot \mathcal{L}[f(t)],\ t_0 > 0$
3. $\mathcal{L}[f(at)](s) = \frac{1}{a}\mathcal{L}[f(t)](\frac{s}{a})$
4. $\mathcal{L}\Big[e^{at} \cdot f(t)\Big](s) = \mathcal{L}[f(t)](s - a)$
5. $\mathcal{L}[f'(t)] = s \cdot \mathcal{L}[f(t)] - f(0^+)$
6. $\mathcal{L}\Big(f''(t)\Big) = s^2 \cdot \overline{f}(s) - s \cdot f(0^+) - f'(0^+)$
7. $\mathcal{L}\Big[f^{(n)}(t)\Big] = s^n \cdot \mathcal{L}[f(t)] - \sum\limits_{k=0}^{n-1} s^{n-k-1} \cdot f^{(k)}(0^+)$
8. $\mathcal{L}\Big[\int_0^t f(\tau)d\tau\Big] = \dfrac{\mathcal{L}[f(t)]}{s}$
9. $\mathcal{L}[t^n f(t))] = (-1)^n \cdot \dfrac{d^n\mathcal{L}[f(t)]}{ds^n}$
10. $\mathcal{L}[f_1 * f_2] = \mathcal{L}[f_1] \cdot \mathcal{L}[f_2]$, gdzie splot oryginałów $(f_1 * f_2)(t) = \int_0^t f_1(\tau) \cdot f_2(t - \tau)d\tau$.

Wzory

$f(t)$	$\mathcal{L}(f)$		$f(t)$	$\mathcal{L}(f)$
$\mathbf{1}(t)$	$\dfrac{1}{s}$		$\sin(\omega t)$	$\dfrac{\omega}{s^2 + \omega^2}$
t^n	$\dfrac{n!}{s^{n+1}}$		$e^{at}\sin(\omega t)$	$\dfrac{\omega}{(s-a)^2 + \omega^2}$
e^{at}	$\dfrac{1}{s-a}$		$\cos(\omega t)$	$\dfrac{s}{s^2 + \omega^2}$
$e^{at}t^n$	$\dfrac{n!}{(s-a)^{n+1}}$		$e^{at}\cos(\omega t)$	$\dfrac{s-a}{(s-a)^2 + \omega^2}$